Nozzle

Flow Through Nozzle and Diffuser [vapour & gases]

Fluids in motion have three forms of energy - pressure, kinetic and potential. For fluids flowing in horizontal ducts or in the case of gases the potential energy term can be ignored. By law of conservation of energy, any increase in pressure energy will result in a decrease in K.E i.e fluid velocity.

Devices which are so designed so as to produce pressure-velocity changes in a fluid are called either Nozzle or Diffuser.

Nozzle → It is a passage of gradually varying cross-section in which the pressure energy and enthalpy [CvT + Pv] of a fluid are converted into kinetic energy.

Application of Nozzles → (i) Steam turbine (impulse type)
~~(ii) Gas Turbine~~
~~(iii)~~ (ii) Turbo jet engine

Diffuser → It is a reverse of nozzle. It is a device in which pressure energy is increased at the expense of kinetic energy.

Application

(i) Rotary compressor [Centrifugal & axial flow]

(ii) Turbojet and Ramjet

Symbols used

P → Fluid Pressure in N/m2 or Pa

v → Specific volume (m3/kg)

ρ → Density (kg/m3)

V → fluid velocity in m/sec

c → Sonic velocity (velocity of sound) in m/sec.

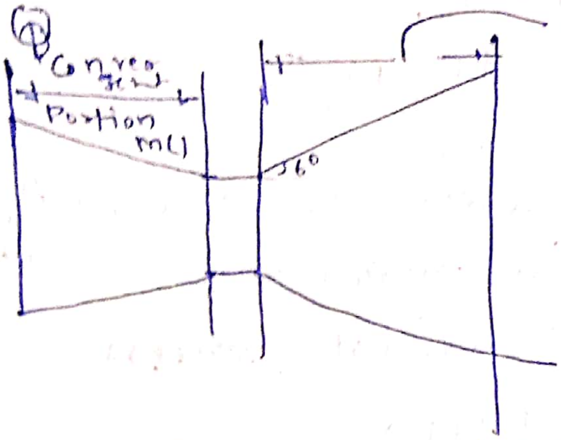
Mach Number (M) → It is ratio of fluid velocity to sonic velocity in the fluid medium. It is an important dimensionless parameter which is used for analyzing compressive flow problem $M = \frac{V}{c}$ $M = \frac{V}{c}$

Nozzle (dA/ is always +ve)

M<1 → dA is −ve i.e area decreases (convergent portion of nozzle)

M=1 → dA is 0 i.e. A has least value this represent Throat of nozzle

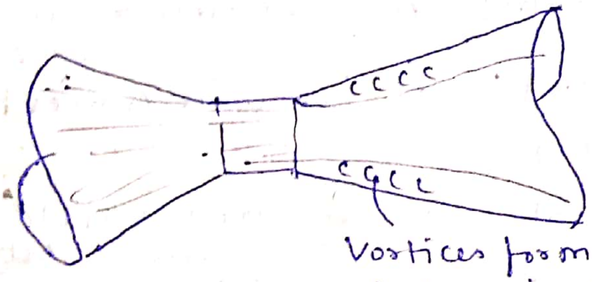
M>1 → dA is +ve i.e area increases (Divergent portion)



$\frac{dA}{A} = \frac{dV}{V}[M^2-1]$

$M = \frac{V}{C}$ = Mach Number

Why divergent portion of Nozzle is made longer



Vortices formed due to separation of flow in diverging cone
[No separation occurs in the Converging cone]

To prevent separation of flow the ~~semi cone~~ angle of semi cone ~~angle of~~ of ~~angle~~ should not be more than 6°. For the given change in dia, from throat to exit, this will require a fairly large length. On the other hand no separation occurs in the converging cone. Hence to reduce frictional losses, it is made of short length.

Formulae for flow through nozzle →

~~Po, Vo~~ Po, Vo be pressure and velocity at entry to nozzle
Vo is called approach velocity

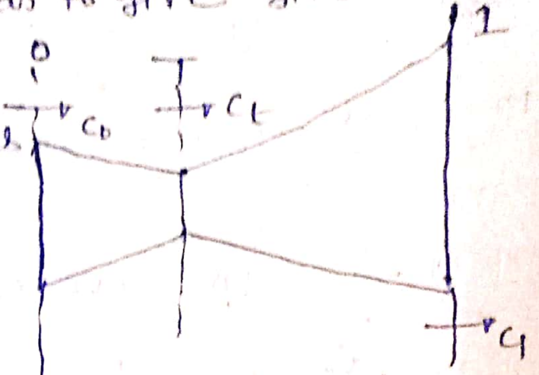
Pt, Vt be the pressure and velocity of fluid at throat

P1, V1 be corresponding quantity at exit

m → mass flow rate through nozzle in kg/sec

Scanned by CamScanner

The Pressure P_0 and P_1 are constant however the throat Pressure P_t can be so chosen as to give give maximum mass flow.



Applying S.T.E.E between sections 0-0 and 1-1

Total energy at 0-0 ± Q ± ws ~~losses~~

= Total energy at 1-1 ± ws + losses

for nozzle flow Q = 0, ws = 0

$$\left(P_0v_0 + C_vT_0 + \frac{V_0^2}{2} + gz_0\right) - \cancel{losses} = P_1V_1 + C_vT_1 + \frac{V_1^2}{2} + gz_1 + losses$$

There is no change in datum energy Z = 0

Neglecting losses [Reversible adiabatic process - isentropic]

$$h_0 + \frac{V_0^2}{2} = h_1 + \frac{V_1^2}{2} \quad (J/kg)$$

$$\frac{V_1^2}{2} - \frac{V_0^2}{2} = (h_0 - h_1) = \Delta h_{is} \rightarrow \text{Isentropic heat drop in nozzle}$$

(If the nozzle is 100% η, it will convert the entire enthalpy into K.E.)

Considering frictional losses. [Irreversible adiabatic]

$$\eta_{Noz} = \frac{\text{Actual heat drop}}{\text{Isentropic heat drop}}$$

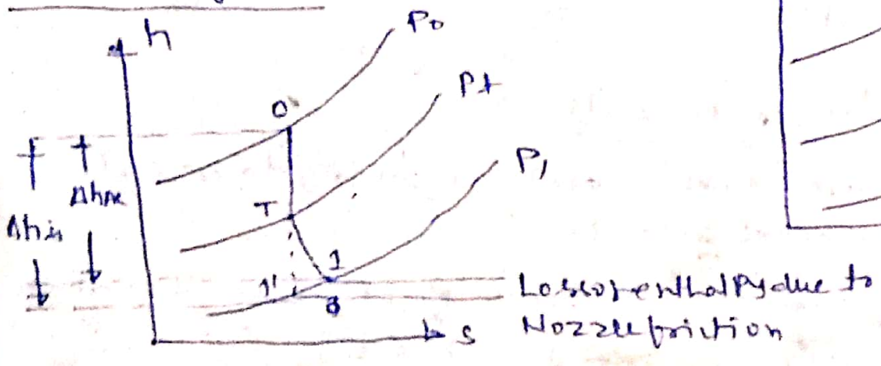
Increase in K.E = η_{noz} × isentropic heat drop.

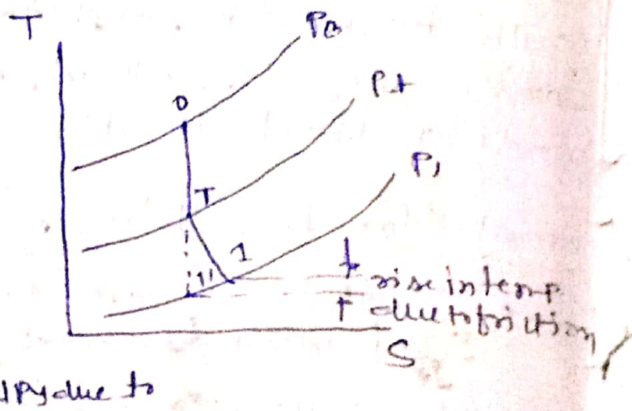
$$\frac{V_1^2}{2} - \frac{V_0^2}{2} = \eta_{noz} \times (h_0 - h_1) = \Delta h_{AC} \; [\text{Actual heat drop in nozzle}]$$

Hence $V_0 \Rightarrow V_1 \sqrt{2 \times \eta_{noz}(h_0 - h_1)}$

[Since the converging portion of nozzle has a short length it will be assumed that the entire losses occurs in diverging portion.]

h-s Diagram





Maximum Mass Flow Rate

P_0, P_1 are constant for a given installation. The value of throat pressure P_t should be so chosen so that mass flow rate is maximum.

$$\dot{m} = \rho_t A_t V_t = \frac{A_t V_t}{v_t}$$

The nozzle converts enthalpy into kinetic energy. Applying energy equation between inlet and throat and assuming 100% efficiency for nozzle.

$$\frac{V_t^2}{2} - \frac{V_0^2}{2} = (h_0 - h_t)$$

Neglecting $V_0^2/2$

$$\frac{V_t^2}{2} = (h_0 - h_t) = c_p[T_0 - T_t]$$

$$= \frac{c_p}{R}[P_0 v_0 - P_t v_t]$$

$$= \frac{\gamma}{\gamma-1} P_0 v_0 \left[1 - \frac{P_t}{P_0} \times \frac{v_t}{v_0}\right]$$

$$V_t^2 = 2\frac{\gamma}{\gamma-1} P_0 v_0 \left[1 - \frac{P_t}{P_0} \times \frac{v_t}{v_0}\right] \quad \text{--- (1)}$$

$Pv =$
$Pv = mRT$
$Pv = RT$
$T = Pv/R$

$c_p - c_v = R$

$\frac{c_p}{c_p} - \frac{c_v}{c_p} = \frac{R}{c_p}$

$1 - \frac{1}{\gamma} = R/c_p$

$\frac{\gamma-1}{\gamma} = R/c_p$

$\frac{c_p}{R} = \frac{\gamma}{\gamma-1}$

$$P_0 v_0^\gamma = P_t v_t^\gamma$$

$$\left(\frac{v_t}{v_0}\right)^\gamma = \left(\frac{P_0}{P_t}\right) \Rightarrow \frac{v_t}{v_0} = \left(\frac{P_0}{P_t}\right)^{1/\gamma}$$

$$= \left(\frac{P_t}{P_0}\right)^{-1/\gamma} \quad \text{--- (2)}$$

$$V_t^2 = 2\frac{\gamma}{\gamma-1} P_0 v_0 \left[1 - \frac{P_t}{P_0} \times \left(\frac{P_t}{P_0}\right)^{-1/\gamma}\right]$$

$$= 2\frac{\gamma}{\gamma-1} P_0 v_0 \left[1 - \left(\frac{P_t}{P_0}\right)^{1-1/\gamma}\right] = 2\frac{\gamma}{\gamma-1} P_0 v_0 \left[1 - \left(\frac{P_t}{P_0}\right)^{\frac{\gamma-1}{\gamma}}\right]$$

from (2) $\quad v_t = v_0 \left(\frac{P_t}{P_0}\right)^{-1/\gamma}$

$$\dot{m} = \frac{A_t V_t}{v_t} = \frac{A_t \sqrt{2\frac{\gamma}{\gamma-1} P_0 v_0 \left[1 - \left(\frac{P_t}{P_0}\right)^{\frac{\gamma-1}{\gamma}}\right]}}{v_0 (P_t/P_0)^{-1/\gamma}}$$

Put $\frac{P_t}{P_0} = x$

$$\dot{m} = \frac{A_t \sqrt{2\frac{\gamma}{\gamma-1} P_0 v_0 \left[1 - (x)^{\frac{\gamma-1}{\gamma}}\right]}}{v_0 (x)^{-1/\gamma}}$$

$$= \frac{A_t}{v_0} \sqrt{2\frac{\gamma}{\gamma-1} P_0 v_0 \left[x^{2/\gamma} - (x)^{\frac{\gamma-1}{\gamma} + 2/\gamma}\right]}$$

$$\dot{m} = \frac{A_t}{v_0} \sqrt{2\frac{\gamma}{\gamma-1} P_0 v_0 \left[x^{2/\gamma} - (x)^{\frac{\gamma+1}{\gamma}}\right]} \quad \text{--- (3)}$$

$x^{2/\gamma} - x^{\frac{\gamma+1}{\gamma}}$ is maximum

i.e. $\frac{d}{dx}\left[x^{2/\gamma} - x^{\frac{\gamma+1}{\gamma}}\right] = 0$

$$\frac{2}{\gamma} x^{2/\gamma - 1} - \frac{\gamma+1}{\gamma} x^{\frac{\gamma+1}{\gamma} - 1} = 0$$

$$\frac{2}{\gamma} x^{2/\gamma - 1} - \frac{\gamma+1}{\gamma} x^{\frac{1}{\gamma}} = 0$$

$$\frac{2}{\cancel{\gamma}} x^{2/\gamma - 1} = \frac{\gamma+1}{\cancel{\gamma}} x^{1/\gamma}$$

$$\frac{2}{\gamma+1} = \frac{x^{1/\gamma}}{x^{2/\gamma - 1}} = x^{\frac{1}{\gamma} - (2/\gamma - 1)} = x^{\frac{1-2+\gamma}{\gamma}} = x^{\frac{\gamma-1}{\gamma}}$$

$$x = \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}} \Rightarrow \boxed{\frac{P_t}{P_0} = \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}}}$$ Critical pressure Ratio.

$$\dot{m} = \frac{A_t}{v_0}\sqrt{2\frac{\gamma}{\gamma-1}\left[x^{2/\gamma} - (x)^{\frac{\gamma+1}{\gamma}}\right] P_0 v_0}$$

$$= \frac{A_t}{v_0}\sqrt{2\frac{\gamma}{\gamma-1}\left[\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1} \times \frac{2}{\gamma}} - \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1} \times \frac{\gamma+1}{\gamma}}\right]}$$

$$= \frac{A_t}{v_0}\sqrt{2\frac{\gamma}{\gamma-1} P_0 v_0 \left[\left(\frac{2}{\gamma+1}\right)^{\frac{2}{\gamma-1}} - \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}}\right]}$$

$$= \frac{A_t}{v_0}\sqrt{2\frac{\gamma}{\gamma-1} P_0 v_0 \left(\frac{2}{\gamma+1}\right)^{\frac{2}{\gamma-1}}\left[1 - \frac{2}{\gamma+1}\right]}$$

$$= \frac{A_t}{v_0}\sqrt{2\frac{\gamma}{\gamma-1} P_0 v_0 \left(\frac{2}{\gamma+1}\right)^{\frac{2}{\gamma-1}}\left(\frac{\gamma-1}{\gamma+1}\right)} = \frac{A_t}{v_0}\sqrt{\gamma P_0 v_0 \left(\frac{2}{\gamma+1}\right)^{\frac{2}{\gamma-1}} \times \left(\frac{2}{\gamma+1}\right)}$$

$$= \frac{A_t}{v_0}\sqrt{\gamma P_0 v_0 \left(\frac{2}{\gamma+1}\right)^{1+\frac{2}{\gamma-1}}} = \frac{A_t}{v_0}\sqrt{\gamma P_0 v_0 \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}}}$$

Fluid in nozzle	Critical pressure ratio $\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}}$
1 Steam [Dry Saturated at entry] $\gamma = 1.135$	$\frac{P_t}{P_0} = 0.58$
(2) Steam [Superheated at entry] $\gamma = 1.3$	0.546
3) Air $\gamma = 1.4$	0.528
(4) Hot gases $\gamma = 1.33$	0.540.